On damaged interfaces in masonry buildings: a multi-scale approach

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Abstract

The aim of the present work is to propose a new micro-mechanical deductive approach to predict effective mechanical properties of micro-cracked brick/mortar interface in masonry structures. This model, based on a previous study proposed by Rekik and Lebon (2010), represents an innovative and analytically consistent way to describe meso-scale damaging phenomenon localized at the interface between deformable solids. The focal point is to assume the existence of a new material layer between brick and mortar, called interphase. This multi-scale method combines standard homogenization technique (law of mixtures, classical bounds, numerical, random), continuum damage mechanics (CDM) and asymptotic technique in order to obtain the brick/mortar interface constitutive law. In this work it will be shown that only soft behavior interface is considered. It has been found that the variable orientation of micro-cracks induces an evolution of the material simmetry. Moreover, the angle influence on interface's stiffness appears only at higher order terms of the asymptotic expansion.

Keywords: interface, asymptotic analysis, masonry, micro-cracks

1. INTRODUCTION

In the last twenty years several authors (Abdelmoula et al., 1998; Lebon and Rizzoni, 2011, 2010; Lebon and Zaittouni, 2010; Lourenco et al., 1997; Pelissou and Lebon, 2009) have established that the interface elements reflect the main interactions occurring between constituents of masonry structures, bricks and mortar indeed. For this reason, various studies, in the general framework of micro-damage-mechanics, have been presented for modeling the behavior of interfaces and predicting their failure modes. Among other, Lotfi and Shing (1994) proposed a model in which the mortar joints were modeled with interface element and masonry units were modeled with smeared crack element (in analogy with smeared crack concrete models). Other authors, like Alfano and Sacco (2006), adopted the strategy to model the brick/mortar and the brick/brick interfaces as cohesive-zone in which damage and friction occur. Some studies, like one of Del Piero and Raous (2010), for example, expressed the constitutive law at the interface in terms of contact traction and the relative displacements of two surfaces interacting at the joint. In their model the fracture of the joint and the subsequent sliding are associated with the interface yield condition. Method based on limit analysis combined with a homogenization technique was shown by De Buhan and De Felice (1997) to be a powerful structural analysis tool, giving accurate collapse predictions.

In this paper, a multi-scale model for micro-cracked interfaces based on CDM and asymptotic techniques is presented. The novelties of the proposed work are the assumption of the existence of a third material inserted between the units and mortar, called interphase, which accounts for the noticeable differences existing between the mechanical properties of

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bricks and mortar and the non-random orientation of micro-cracks families which makes able to take into account the anisotropy induced by micro-cracks.

2. GENERALITIES OF THE MODEL

Generally in literature, masonry units have been discretized using continuum elements, whereas mortar joints have been modeled in the form of weakness planes using interface elements. The main limitation of the, so-called, simplified micro-modeling approach, is the fact that the interactions between joints and brick units cannot be satisfactorily described. The proposed model focuses on the behavior of masonry structures at the meso-scale level. The most original feature of this kind of approach, firstly proposed by Rekik and Lebon (2010, 2012), is that it includes a third material inserted between the units and mortar, called interphase, which accounts for the noticeable differences existing between the mechanical properties of bricks and mortar. The novelty of the proposed model is taking into account the variable orientation of micro-cracks with respect to the masonry orthotropy axes.

In order to model the meso-scale damage processes in masonry structures, which according to experimental evidences (Fouchal et al., 2009; Gabor et al., 2006), are localized at brick/mortar interface level, the present three-steps method was developed. In the general framework of the synergistic damage mechanics approach (Talreja, 2006) the material behavior of interphase is characterized at the micro-scale.

2.1. *Stratified composite homogenization*

After the choice of a representative volume element of brick/mortar interphase, the effective properties of the crack-free material are obtained using homogenization techniques for laminate composites. For the sake of simplicity both constituents are assumed to be isotropic and elastic materials with the same volume fraction. In the compliance form, their constitutive law reads:

$$\varepsilon^{\zeta}_{ij} = S^{\zeta}_{ijkl}\sigma^{\zeta}_{ik} = \frac{1+\nu^{\zeta}}{E^{\zeta}}\sigma^{\zeta}_{ij} - \frac{\nu^{\zeta}}{E^{\zeta}}\sigma^{\zeta}_{kk}\delta_{ij} \qquad (1)$$

where S^{ζ}, E^{ζ} and ν^{ζ} are respectively the compliance tensor, the Young's modulus and the Poisson ratio of phase ζ ($\zeta = b$ for the brick, $\zeta = m$ for the mortar). The behaviour law of the laminate brick/mortar is written as follow:

$$\begin{aligned} \bar{\varepsilon} = \bar{S}^0 : \bar{\sigma} \quad \text{where} \quad \bar{\varepsilon} &= \sum_{\zeta=b,m} f^{\zeta}\varepsilon^{\zeta} \\ &= \sum_{\zeta=b,m} f^{\zeta} S^{\zeta} : \sigma^{\zeta} \end{aligned} \qquad (2)$$

where f^{ζ} denotes the volume fractions of phase ζ and $\bar{S}^0$ is the effective fourth-order compliance tensor of the homogeneous equivalent crack-free material supposed to be transversely isotropic. The five independent coefficients of the compliance tensor $\bar{S}^0$ are determined by applying three independent loads. The same results may be found in Boutin (1996), where the macroscopic law of the laminate is given in the stiffness form $\bar{C}^0 = \left(\bar{S}^0\right)^{-1}$.

The model is developed assuming the plane stress hypothesis in direction e_2 on the effective material, so that the 3D problem is reduced to a 2D problem in the (e_1, e_3) plane.

2.2. *Micro-cracked material homogenization*

In this section, it is assumed that the previously homogenized material contains an arbitrary distribution of rectilinear cracks of density ρ located on the plane (e_1, e_3) in a representative area $A = L_0\varepsilon$, where L_0 is the bed mortar length and ε is the interphase thickness. Mauge and Kachanov (1994) and Tsukrov and Kachanov (1998) provided an accurate approximation of the effective behavior of such a material for open cracks; the average strain $\bar{\varepsilon}$ in a solid with N families of micro-cracks can be written in the form:

$$\bar{\varepsilon} = \bar{S} : \bar{\sigma} = (\bar{S}^0 + \Delta\bar{S}) : \bar{\sigma} \qquad (3)$$

where $\bar{S}$ (resp. $\bar{S}^0$) is the effective compliance of the cracked (resp. crack-free) material and

$$\begin{aligned} \Delta\bar{S}_{ijhl} = &\frac{1}{2A}\sum_{k=1}^{N}[n^{(k)}_i n^{(k)}_h B_{jl} + n^{(k)}_i n^{(k)}_l B_{jh} + \\ &+ B_{ih} n^{(k)}_j n^{(k)}_l + B_{il} n^{(k)}_j n^{(k)}_h](l^{(k)})^2 \end{aligned} \qquad (4)$$

is the anisotropy induced by the presence of the micro-cracks in which $2l^{(k)}$ and $n^{(k)} = (-sin\phi^{(k)}, 0, cos\phi^{(k)})$ are the mean length and the normal unit vector of the k^{th} family of cracks and A is the area of the representative 2D-domain. One define the angle $\phi = \widehat{(e_1,t)}$ between the unit tangent vector of micro-cracks and the matrix orthotropy axes (Figure 1). The second rank symmetric tensor $B^{(k)}$

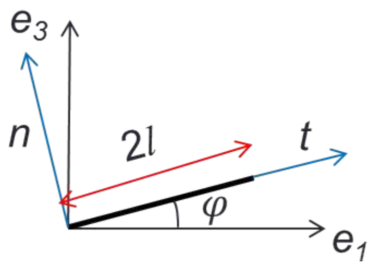

Figure 1: *Local cracks vectors and matrix orthotropy axes*

can be called the crack compliance tensor of the k^{th} family of cracks, which depends on the anisotropy of the virgin material (Tsukrov and Kachanov, 2000). It is defined as $B = B_{tt}(e_1 \otimes e_1) + B_{nn}(e_3 \otimes e_3)$ with $B_{tt} = C(1-D)$ and $B_{nn} = C(1+D)$, remark that it is independent of crack orientation ϕ. C and D are constants which only depend on the virgin material properties:

$$\begin{cases} C = \frac{\pi}{4}\frac{\sqrt{E_1^0}+\sqrt{E_3^0}}{\sqrt{E_1^0 E_3^0}}\left(\frac{1}{G_{13}^0} - 2\frac{\nu_{13}^0}{E_1^0} + \frac{2}{\sqrt{E_1^0 E_3^0}}\right)^{\frac{1}{2}} \\ D = \frac{\sqrt{E_1^0}-\sqrt{E_3^0}}{\sqrt{E_1^0}+\sqrt{E_3^0}} \end{cases} \quad (5)$$

E_1^0, E_3^0, ν_{13}^0 and G_{13}^0 are the effective elastic engineering constants of the crack-free material. These constants are easily derived from the effective elastic compliances of the crack-free material $\bar{S}^0$. One can remark in (5) the dependence of the sign of D on the anisotropy ratio E_1^0/E_3^0.
A single family of parallel micro-cracks with mean length $2l$ and orientation ϕ is considered for the numerical validation and the proper scalar crack density parameter in 2D case is $\rho = 1/A\sum l^2$.
Thus, under plane stress conditions in the $(e_1,\ e_3)$ plane the compliance matrix of micro-cracked material $\bar{S}$ in classical Voigt notation reads:

$$\begin{pmatrix} \frac{1}{E_1}+2B_{tt}\rho sin^2\phi & -\frac{\nu_{13}}{E_1} & -\frac{1}{2}B_{tt}\rho sin2\phi \\ -\frac{\nu_{13}}{E_1} & \frac{1}{E_3}+2B_{nn}\rho cos^2\phi & -\frac{1}{2}B_{nn}\rho sin2\phi \\ -\frac{1}{2}B_{tt}\rho sin2\phi & -\frac{1}{2}B_{nn}\rho sin2\phi & \frac{1}{G_{13}}+\frac{1}{2}\rho\left(B_{tt}cos^2\phi+B_{nn}sin^2\phi\right) \end{pmatrix}$$

By inverting compliance matrix one obtain the elasticity matrix and in particular the elastic constants C_{33}, C_{55} and C_{35}, which represent the stiffnesses in normal and tangent-to-the-interface directions and the coupling stiffness.

2.3. *Asymptotic analysis and interface law*

In this section, the behavior law of the interface is obtained. First of all, it is considered a thin interphase constituted of the material defined in the previous section, sandwiched between brick and mortar. It is taken ε to denote the thickness of this joint, which is assumed to be constant in e_1 direction. Since the interphase is thin and soft, it is chosen to use asymptotic techniques to study the limit problem (Figure 2). One takes $\bar{C}$ to denote the elasticity tensor of the joint and the limits

$$\lim_{\varepsilon\to 0} \bar{C}_{ijkl}/\varepsilon$$

are assumed to exist.
The asymptotic expansions (denoted with $\hat{C}_{ik}$) of

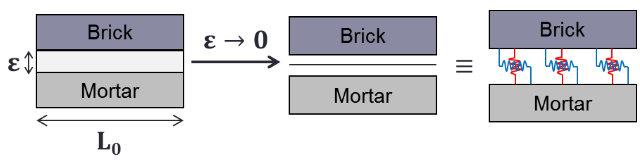

Figure 2: *Asymptotic technique*

the elastic constants C_{33}, C_{55} and C_{35} defined above, read as:

$$\begin{aligned} \hat{C}_{33} &= \hat{C}_{33}^0 + \hat{C}_{33}^1\varepsilon + O(\varepsilon^2) \\ \hat{C}_{35} &= \hat{C}_{35}^0 + \hat{C}_{35}^1\varepsilon + O(\varepsilon^2) \\ \hat{C}_{55} &= \hat{C}_{55}^0 + \hat{C}_{55}^1\varepsilon + O(\varepsilon^2) \end{aligned} \quad (6)$$

Finally, the thin interphase is replaced by an interface with its *soft* constitutive law defined along the limit surface: $\sigma_{i3} = \bar{C}_{i3i3}\,[u_i]$ with $i = 1,3$ where [ ] denotes the jump along S_ε. Using expressions (6) and writing the crack density scalar in the form: $\rho = l^2/\varepsilon L_0$, one obtains:

$$\begin{aligned} \hat{C}_{33}(\varepsilon) &= \frac{L_0\varepsilon}{2C(1+D)l^2} + O(\varepsilon^2) \\ \hat{C}_{35}(\varepsilon) &= 0 \\ \hat{C}_{55}(\varepsilon) &= \frac{2L_0\varepsilon}{C(1-D)l^2} + O(\varepsilon^2) \end{aligned} \quad (7)$$

It is found an interface law which links the stress vector to the jump displacement vector via a diagonal matrix. It is important to remark that this is a simplified choice to reduce the continuous model of the interphase material to a simple mechanistic model obtained considering springs in the normal and tangential direction, which stiffnesses are:

$$\begin{array}{l} \bar{C}_N = \frac{L_0}{2C(1+D)l^2} \\ \bar{C}_{TN} = 0 \\ \bar{C}_T = \frac{2L_0}{C(1-D)l^2} \end{array} \tag{8}$$

where N and T denote, respectively, the normal and the tangential directions with respect to the interface. In this model the angle ϕ can have, for geometrical reason, randomly value in the range $0 \leq \phi \leq p\varepsilon$ with p a positive scalar. The resulting interface's stiffnesses are in agreement with those of the Rekik-Lebon model, that is the terms at 0-order are equal to zero and a *soft* interface behavior is obtained, in which the dependency on the micro-cracks angle disappears. One can prove that the angle ϕ influences only the terms at order higher then one.

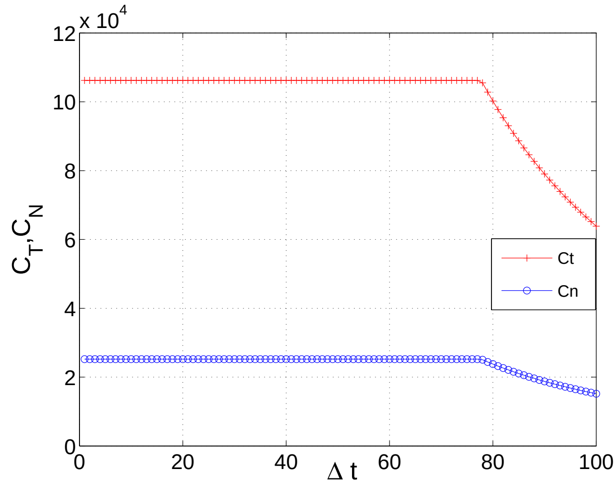

Figure 3: *Variation of the overall elastic coefficients C_N and C_T with respect to time*

2.4. *Damage evolution*

The present model takes the evolution of the micro-crack into account by taking a variable crack half length l depending on the load. It is assumed that the half-length l depends only on the predominant tangential stress τ by neglecting its dependence on the normal stress. A bilinear evolution law is considered (Figure 4). It is assumed that l remains constant $l = l_c$ until a certain value τ_c of the shear stress has been reached. This first phase corresponds to a stable state of the material in which crack propagation occurs. From this value, the crack half length l evolves linearly with respect to the shear stress τ up to a second value of the crack length l_u reached at the maximum shear stress value τ_u. This linear evolution represents the phase of crack propagation, which leads to the failure of the interface.

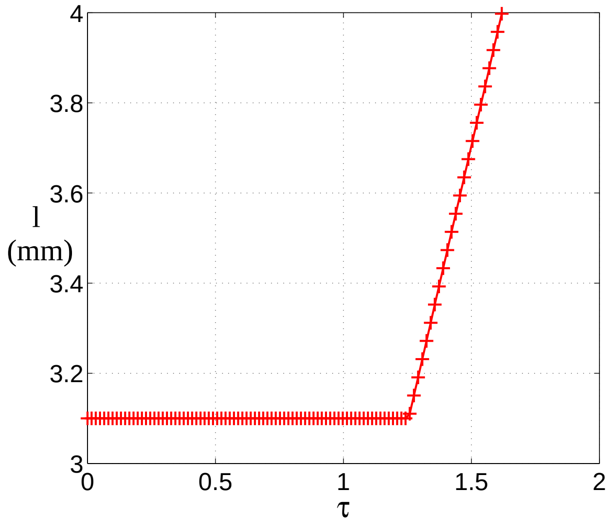

Figure 4: *Bilinear evolution law*

2.5. *Identification process and numerical results*

The model is implemented in a finite element software. A plane stress modeling is pursued using a regular mesh of $Q4$ elements for the overall volume. For interface elements, 0-thickness $Q4$ elements are used. An incremental explicit algorithms is used to solve the local problem. The parameters τ_c, τ_u, l_c and l_u of the evolution law, result from an *identification* process based on experimental data, see Table 1. The experiments (Fouchal et al., 2009) considered for the identification are simple shear tests on small assemblages of three full bricks; these samples were subjected to a monotonously increasing load until damage occurred (Figure 5). Experimental data of two tests are considered. In Figures 6 and 7 the results

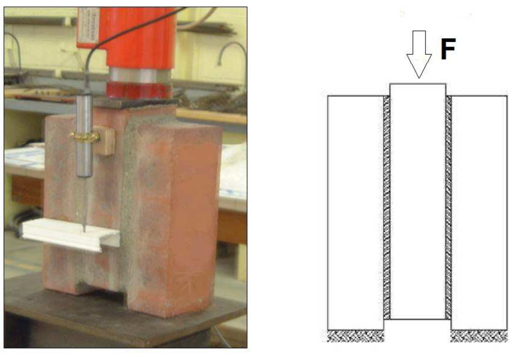

Figure 5: *Experimental test*

in terms of “load-displacements” curves for numerical and experimental cases are compared, showing a good correlation.

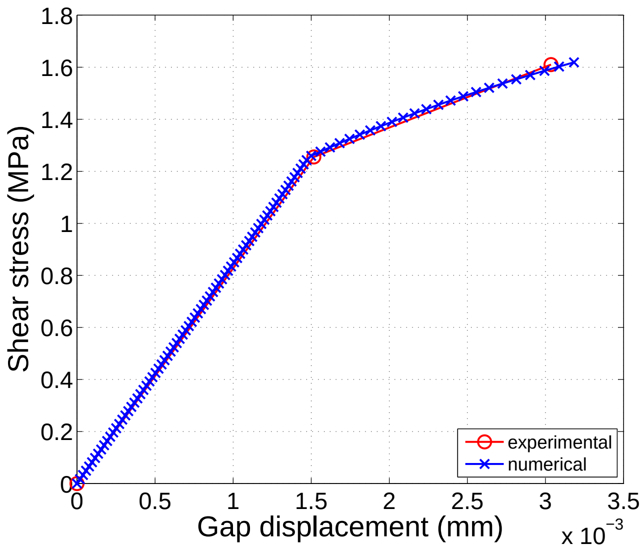

Figure 6: *Identification of the parameters of the evolution law, by comparison with experimental data (P1, Fouchal et al. (2009))*

3. CONCLUSIONS

In this work a multi-scale model for micro-cracked interfaces in masonry structures based on continuum damage mechanic and asymptotic techniques is presented. The novelties of the proposed model are the

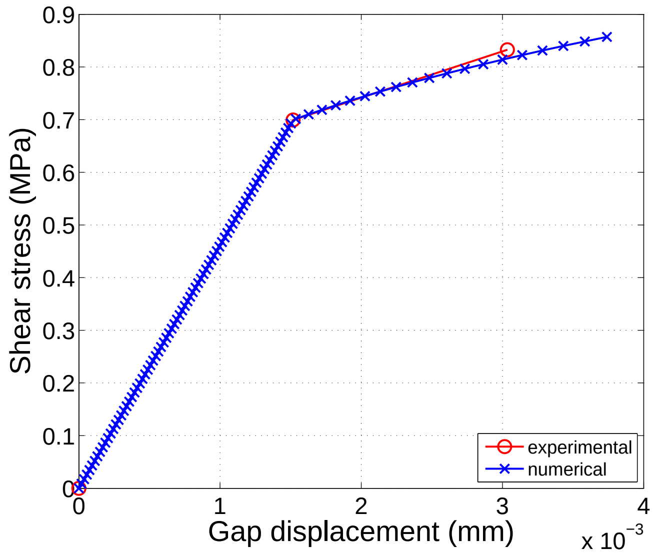

Figure 7: *Identification of the parameters of the evolution law, by comparison with experimental data (P4, Fouchal et al. (2009))*

	test P1	test P4
$\tau_c(MPa)$	1.26	0.7
$\tau_u(MPa)$	1.62	0.86
$l_c(mm)$	3.1	4.2
$l_u(mm)$	4.0	6.0

Table 1: Parameters of the evolution law obtained by the identification process

assumption of the existence of an interphase material inserted between the units and mortar, which accounts for the differences existing between the mechanical properties of the constituents and the variable orientation of micro-cracks families which makes able to take into account the anisotropy induced by micro-cracks. Finally, the interface behavior law is obtained. The model takes into account the evolution of the damage caused by micro-cracks propagation thanks to a few-parameters bilinear law between the crack half length l and the shear load applied. A validation process is performed to identify the law's parameters, showing how a quite simple evolution law is able to accurately describe the response of a small masonry assemblage subjected to pure shear.
The future project is to improve the interface model

taking into account the geometrical non-linearities, like roughness and some material non-linearities as the opening-closure effect.

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